Conformal gravity one-loop partition function in four dimensions

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Outline

- Introduction and motivation
- Conformal gravity
- One-loop partition function
- Summary and discussion

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- has well defined variational principle and finite response functions

Grumiller et al. 2013

Conformal gravity

$$S_{CG} = \int_{\mathcal{M}} d^4x \sqrt{|g|} C^{\lambda}{}_{\mu\sigma\nu} C_{\lambda}{}^{\mu\sigma\nu}$$

Conformal gravity

conformal gravity action

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Conformal gravity

Weyl tensor

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$$C_{\mu\nu\lambda\sigma} = R_{\mu\nu\lambda\sigma} - \left(g_{\mu[\lambda} R_{\sigma]\nu} - g_{\nu[\lambda} R_{\sigma]\mu}\right) + \frac{1}{3} R g_{\mu[\lambda} g_{\sigma]\nu}$$

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Weyl factor

One loop partition function

Correction to partition function, correction to TD quantities

$$Z_{one-loop} = \int \mathcal{D}\delta g_{\mu\nu} \times ghost \times exp(-\delta^{(2)} S)$$

determinants obtained upon eliminating gauge degrees of freedom

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determinants obtained upon eliminating gauge degrees of freedom

transverse traceless decomposition of metric

ghost determinant

One loop partition function

ghost

$$Z_{CG}^{(4)} = \frac{\left[\det(4\Lambda + \nabla^2)_0\right]^{1/2}\left[\det(-3\Lambda - \nabla^2)_1\right]^{1/2}}{\left[\det(-4\Lambda + \nabla^2)_2\right]^{1/2}\left[\det(-2\Lambda + \nabla^2)_2\right]^{1/2}}$$

One loop partition function

contribution from the conformal ghost

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contribution from PMR

One loop partition function

EG part

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We evaluate the partition function using the heat kernel and group theoretic approach

One loop partition function: group theoretic method

$$-\ln\det(-\Delta_{(S)}+m_S^2)=\sum_{k\in\mathbb{Z}_+}\chi_{(s,0)}^{SO(2l+1)}\frac{2}{(1-e^{-k\beta})^{2l+1}e^{k\beta l}e^{\frac{k\beta}{2}}}\frac{1}{k}e^{-k\beta\sqrt{\rho^2+s+m_S^2}}$$

One loop partition function: group theoretic method

Character of SO(2l+1) representation

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One loop partition function: group theoretic method

inverse temperature

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$\Downarrow$

$$q\equiv e^{-\beta}$$

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$$q\equiv e^{-\beta}$$

$$\ln Z_{CG}^{(4)}=-\sum_{k\in\mathbb{Z}_+}\frac{q^{2k}(-5+4q^{2k}-5q^k)}{(1-q^k)^3k}$$

Summary and Discussion

- The one-loop partition function of conformal gravity in four dimensions keeps the expected structure of CG partition functions: it consists of the contribution from conformal ghost, partially massless response and Einstein gravity part.

- analysis of CG one loop partition function on AdS_4 background compared to computation on $\mathbb{R} \times S^3$

Thank you!

AdS/CFT and the role of partition function

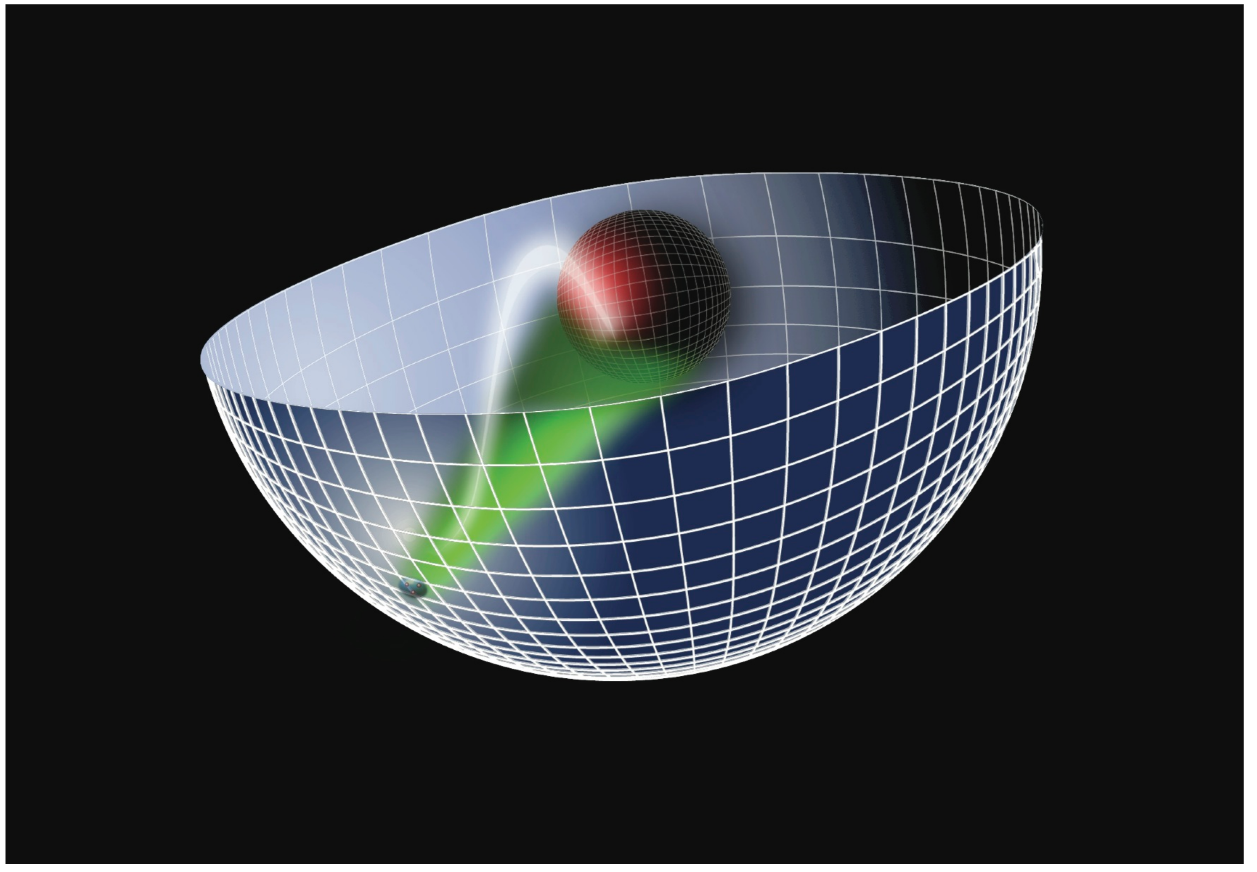
- We consider CG in the Anti-de Sitter/conformal field theory framework

AdS $\longrightarrow$ CFT

$$\phi_0(x) = \phi(\rho = 0, x) = \phi_{\partial AdS(x)}$$

$$Z_{QFT}[\phi_0] = Z_{gravity}[\phi \to \phi_0]$$

$\Rightarrow$ perturbative approach



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- semi-classical limit
- saddle point approximation